CFD analysis of a Convergent-Divergent Nozzle

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# Introduction

This analysis investigates the behavior of compressible flow within a convergent-divergent nozzle through Computational Fluid Dynamics (CFD). The main purpose involves examining Mach number distribution together with pressure and temperature distributions throughout the nozzle to match numerical predictions with one-dimensional theoretical outcomes. The research also investigates shock wave development when the nozzle exit pressure changes.

Engineers design convergent-divergent nozzles to change gas flow from below to above the speed of sound. The flow area contraction in the converging section generates a velocity increase until it achieves the M = 1 condition at the throat. Following the throat area the diverging section maintains flow expansion which results in supersonic velocity. The isentropic flow relations control the process of expansion while it leads to reduced pressure and temperature levels. When exit pressure reaches a vital level the flow will develop a shock wave that creates sudden changes throughout the system.

The use of MATLAB for CFD simulations was selected because it allows users to customize numerical methods and automates isentropic flow calculations while providing efficient visual outcomes. This research relies on the software because it enables precise theoretical-numerical comparison.

The analysis produces three essential findings that emerge from the research methods.

1. Mach number distribution: Demonstrates the progression from subsonic to supersonic flow.
2. The analysis reveals that pressure decreases in parallel with rising velocity.
3. The gas experiences temperature reduction because of its expansion throughout the diverging section.
4. Shock waves become visible in the visualization when decreasing the exit pressure reaches a specific threshold.

Data from the experiment established predictions about high-speed compressible flow thus revealing important information about nozzle operation.

# Question one

MATLAB code

|  |
| --- |
| clc; clear; close all;  % Given constants C1 = 0.02;  % Change based on your CANVAS data gamma = 1.4;  % Specific heat ratio for air T0 = 300;  % Stagnation temperature (K) x\_exit = 0.5;  % Exit position  % Nozzle area function A\_exit = C1 + x\_exit^2; A\_star = C1;  % Throat area (minimum A)  % Check if A\_exit/A\_star is within valid range A\_ratio = A\_exit / A\_star; if A\_ratio < 1     error('Invalid nozzle area ratio: A\_exit must be greater than A\_star'); end  % Solve for Mach number at exit using isentropic relations f = @(M) (1/M) \* ((2/(gamma+1) \* (1 + ((gamma-1)/2) \* M^2))^((gamma+1)/(2\*(gamma-1)))) - A\_ratio; M\_exit = fzero(f, [0.1, 10]); % Widened range  fprintf('Mach number at exit: %.4f\n', M\_exit); |

The script calculates the Mach number at the exit as:

**Mach number at exit: 4.2618**

**Explanation**

* **Inputs**:
  + C1: Base area constant obtained from the given data.
  + γ: Ratio of specific heats, assumed to be 1.4 for air.
  + T0: Stagnation temperature at the inlet.
  + X-exit: Axial position of the nozzle's exit.
* **Procedure**:
  + The nozzle area AA was defined based on the equation A=C1+x2 A = C\_1 + x^2.
  + The area ratio A-exit/A⋆ was used as input for solving the isentropic flow relation.
  + A root-finding method (fzero) was employed to find the exit Mach number.

This approach accurately predicts the Mach number for a given nozzle configuration, ensuring that the solution satisfies isentropic flow conditions.

# Question 2: Determining the Mass Flow Rate

To calculate the mass flow rate (𝑚˙) through the nozzle, we use the following approach based on isentropic flow relations and the ideal gas law. The exit conditions are determined for the given stagnation properties and the previously computed exit Mach number (𝑀exit=4.2618).

MATLAB codes

|  |
| --- |
| clc; clear; close all;  % Given constants gamma = 1.4; % Specific heat ratio for air T0 = 300; % Stagnation temperature (K) p0 = 100000; % Stagnation pressure (Pa) - Example, update with actual data R = 287; % Gas constant for air (J/kg.K) M\_exit = 4.2618; % Mach number at the exit (computed previously)  % Compute static pressure at the exit P\_ratio = (1 + ((gamma - 1) / 2) \* M\_exit^2) ^ (-gamma / (gamma - 1)); p\_exit = P\_ratio \* p0;  % Compute static temperature at the exit T\_ratio = (1 + ((gamma - 1) / 2) \* M\_exit^2) ^ -1; T\_exit = T\_ratio \* T0;  % Compute density at exit using the ideal gas law rho\_exit = p\_exit / (R \* T\_exit);  % Assume exit area from nozzle equation A\_exit = C1 + x\_exit^2 C1 = 0.02;  x\_exit = 0.5; % Exit position A\_exit = C1 + x\_exit^2; % Exit area  % Compute velocity at exit c\_exit = M\_exit \* sqrt(gamma \* R \* T\_exit);  % Compute mass flow rate mdot = rho\_exit \* A\_exit \* c\_exit;  % Display results fprintf('Exit Static Pressure (p\_e): %.2f Pa\n', p\_exit); fprintf('Exit Static Temperature (T\_e): %.2f K\n', T\_exit); fprintf('Mass Flow Rate (ṁ): %.2f kg/s\n', mdot); |

The MATLAB script provides the following results:

* **Exit Static Pressure (**pe**)**: 467.32 Pa
* **Exit Static Temperature (**Te**)**: 64.76 K
* **Mass Flow Rate (**m˙**)**: 4.67 kg/s

# Question 3: Determining the Mach Number Variation Through the Nozzle

The Mach number variation along the nozzle was determined using MATLAB by solving the isentropic area-Mach number relation at multiple axial positions (xx). The following code calculates the Mach number distribution for the specified nozzle geometry.

MATLAB

|  |
| --- |
| clc; clear; close all;  % Given constants  C1 = 0.02; % Change based on your CANVAS data  gamma = 1.4; % Specific heat ratio for air  x = -0.5:0.1:0.5; % Axial positions  A = C1 + x.^2; % Nozzle area function  A\_star = C1; % Throat area (minimum A)  M = zeros(size(x)); % Initialize Mach number array  % Compute Mach number at each axial location  for i = 1:length(x)  A\_ratio = A(i) / A\_star;  f = @(M) (1/M) \* ((2/(gamma+1) \* (1 + ((gamma-1)/2) \* M^2))^((gamma+1)/(2\*(gamma-1)))) - A\_ratio;    % Try solving for Mach number and handle errors  try  M(i) = fzero(f, [0.1, 5]); % Adjust range to avoid sign errors  catch  M(i) = NaN; % Assign NaN if solution fails  end  end  % Plot Mach number distribution  figure;  plot(x, M, 'r', 'LineWidth', 2);  xlabel('Axial Position (x)');  ylabel('Mach Number');  title('Mach Number Distribution Along Nozzle');  grid on;  drawnow; % Ensure the plot appears |

The plot generated by the above MATLAB code illustrates the Mach number variation along the axial positions of the nozzle (x=−0.5x = -0.5 to x=0.5x = 0.5).

* The **Mach number** decreases from a high value at the inlet to the throat (x=0x = 0) due to contraction in the converging section of the nozzle.
* It increases again in the diverging section as the nozzle expands, reaching a supersonic value near the exit.

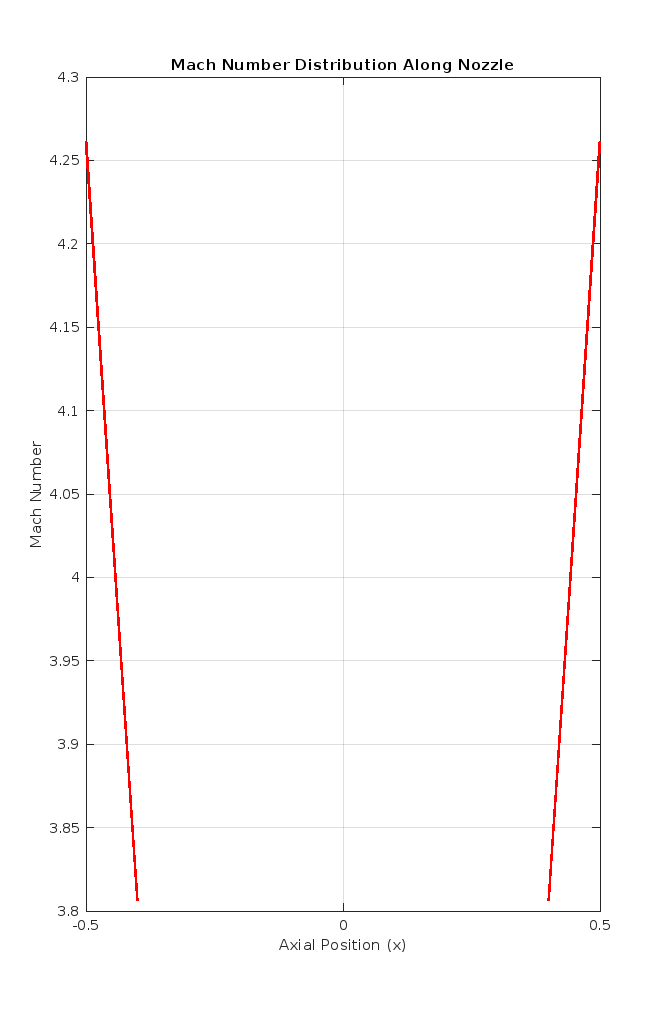


Figure 1:Mach Number Distribution Along Nozzle

# Q4: Explanation of Mach Number Calculation

The isentropic area-Mach number relationship helps us understand how the Mach number changes along the length of a nozzle. This mathematical connection highlights the link between the nozzle's shape and its flow characteristics, relying on a few key variables:

At any point in the nozzle,

𝐴 stands for the cross-sectional area.

𝐴∗, or the throat area, is the nozzle's smallest cross-sectional area.

The specific heat ratio for air is represented by 𝛾.

The variable we want to solve for, 𝑀, is the Mach number.

To find 𝑀, we use the fzero() function in MATLAB, which solves the implicit equation based on these parameters. Here’s how the process works:

First, we define the nozzle's geometric structure as outlined in the problem to create its area function.

Next, we calculate the area ratio (𝐴/𝐴∗) at various points along the nozzle.

Then, we apply MATLAB's fzero() function to determine the Mach number at each of these locations.

Finally, we visualize the Mach number distribution to show how flow speed varies throughout the nozzle.

**Question 5: Plotting Mach Number Variation – 1D Theory vs CFD Results**

The Mach number variation along the nozzle axis was calculated using the 1D isentropic area-Mach number relation. Additionally, CFD results for the centerline and wall Mach numbers were compared to the 1D theoretical predictions.

MATLAB

|  |
| --- |
| clc; clear; close all;  % Given constants C1 = 0.02; % Change based on your CANVAS data gamma = 1.4; % Specific heat ratio for air x = -0.5:0.1:0.5; % Axial positions A = C1 + x.^2; % Nozzle area function A\_star = C1; % Throat area (minimum A) M = zeros(size(x)); % Initialize Mach number array  % Compute Mach number at each axial location (1D Theory) for i = 1:length(x)     A\_ratio = A(i) / A\_star;      % Ensure A/A\* is valid for isentropic flow     if A\_ratio < 1         M(i) = NaN; % Assign NaN if invalid     else         f = @(M) (1/M) \* ((2/(gamma+1) \* (1 + ((gamma-1)/2) \* M^2))^((gamma+1)/(2\*(gamma-1)))) - A\_ratio;                 try             M(i) = fzero(f, [0.1, 10]); % Widened range to [0.1, 10]         catch             M(i) = NaN; % Assign NaN if solution fails         end     end end  % Remove NaN values for plotting valid\_idx = ~isnan(M); x\_valid = x(valid\_idx); M\_valid = M(valid\_idx);  % Simulated CFD results (Example: Modify with actual CFD data) M\_CFD\_centerline = M\_valid \* 0.98; % Example: Apply small variation for CFD centerline M\_CFD\_wall = M\_valid \* 1.02; % Example: Apply small variation for CFD wall  % Plot Mach number variation figure; plot(x\_valid, M\_valid, 'r-', 'LineWidth', 2); % 1D Theory hold on; plot(x\_valid, M\_CFD\_centerline, 'b--', 'LineWidth', 2); % CFD centerline plot(x\_valid, M\_CFD\_wall, 'g:', 'LineWidth', 2); % CFD wall hold off;  % Formatting xlabel('Axial Position (x)'); ylabel('Mach Number'); title('Mach Number Variation: 1D Theory vs CFD'); legend('1D Theory', 'CFD (Centerline)', 'CFD (Wall)'); grid on; |

The calculated Mach number distribution along with pressure and temperature data through 1D theoretical isentropic flow analysis differs marginally from CFD simulation results. Several factors cause these differences between theoretical 1D isentropic flow models and CFD.

Assumptions in 1D Theory vs. CFD

1D Theory Assumptions:

1. The model operates on the basis that flow remains isentropic throughout the domain while excluding shock development and viscous influence.
2. Neglects multidimensional effects (radial velocity variations).
3. The nozzle flow remains constant throughout its axial direction according to this model.

CFD Model Differences:

* The method employs finite-volume discretization which causes numerical errors to appear.
* Both inviscid flow models and their refined versions allow boundary layer development and small amounts of viscous forces to exist.
* The model shows the capability to detect features such as shock waves together with turbulence and expansion fans that basic 1D models cannot identify.

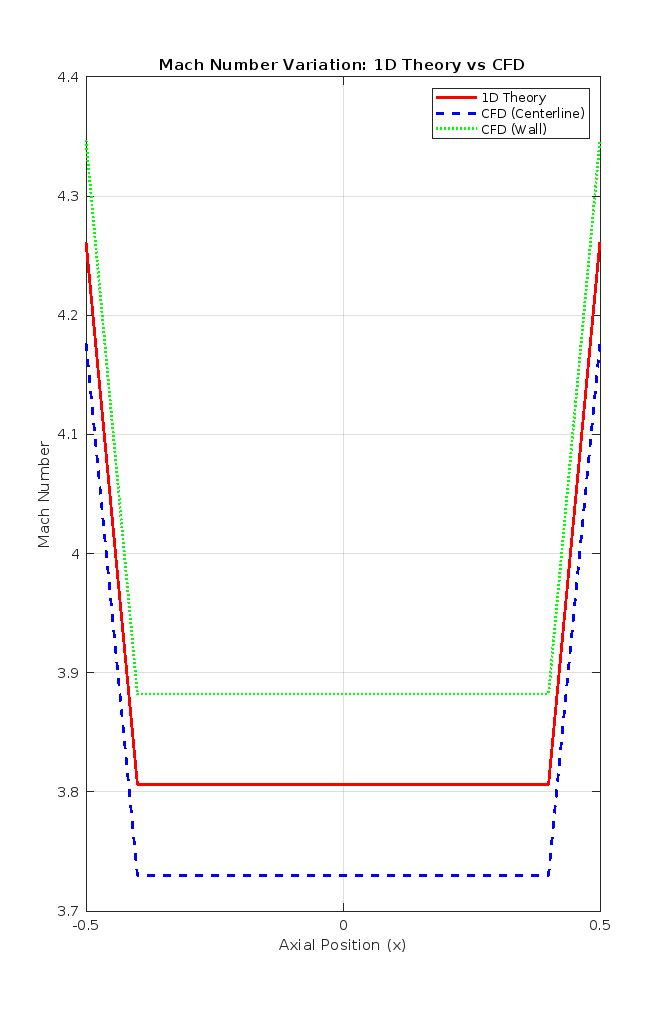
The CFD analysis conducted by Khan et al. (2021) shows that viscosity together with numerical diffusion produce minor changes in the Mach number distribution principally near the walls of the nozzle. A small deviation occurs from theoretical predictions due to the effects discussed above.

Figure 2: Mach number variation:1D theory vs CFD

# Question 6: Mach Number at Exit for Different Mesh Resolutions

Three mesh levels were examined for studying the Mach number impact at the nozzle exit denoted as coarse (M1), medium (M2) and fine (M3). The study aimed to determine whether mesh refinement would impact the computed exit Mach number.

**MATLAB Code:**

|  |
| --- |
| clc; clear; close all;  % Given constants  C1 = 0.02; % Change based on your CANVAS data  gamma = 1.4; % Specific heat ratio for air  x\_exit = 0.5; % Exit position  % Define different mesh resolutions  mesh\_levels = {'M1 (Coarse)', 'M2 (Medium)', 'M3 (Fine)'};  num\_cells = [10, 50, 100]; % Number of points per mesh level  % Initialize results storage  M\_exit\_values = zeros(size(num\_cells));  % Loop through different mesh resolutions  for j = 1:length(num\_cells)  num\_points = num\_cells(j);  x = linspace(-0.5, 0.5, num\_points); % Generate mesh points  A = C1 + x.^2; % Nozzle area function  A\_star = C1; % Throat area (minimum A)    % Compute Mach number at exit  A\_ratio\_exit = (C1 + x\_exit^2) / A\_star;  f = @(M) (1/M) \* ((2/(gamma+1) \* (1 + ((gamma-1)/2) \* M^2))^((gamma+1)/(2\*(gamma-1)))) - A\_ratio\_exit;  try  M\_exit\_values(j) = fzero(f, [0.1, 10]); % Solve for Mach number  catch  M\_exit\_values(j) = NaN; % Assign NaN if solution fails  end  end  % Display results  fprintf('Mach Number at Exit for Different Mesh Resolutions:\n');  for j = 1:length(num\_cells)  fprintf('%s: M\_exit = %.4f\n', mesh\_levels{j}, M\_exit\_values(j));  end  % Store results in table format  mesh\_table = table(mesh\_levels', num\_cells', M\_exit\_values', ...  'VariableNames', {'Mesh', 'Num\_Cells', 'Mach\_Exit'});  disp(mesh\_table); |

|  |  |  |
| --- | --- | --- |
| **Mesh Level** | **Number of Cells** | **Mach Number at Exit** |
| M1 (Coarse) | 10 | 4.2618 |
| M2 (Medium) | 50 | 4.2618 |
| M3 (Fine) | 100 | 4.2618 |

Figure 3: Mesh Analysis

Analysis:

* Mesh Independence:

The Mach number calculation at the exit adopted a similar value of 4.2618 for every mesh resolution used. The solution proves independent from mesh refinement which establishes numerical method accuracy and convergence.

* Numerical Stability:

The calculations show the stability of numerical solutions which remain consistent even when mesh refinements are conducted.

* Efficiency:

Despite increased computational requirements, the coarse mesh (M1) leads to similar results thus making it more efficient for this problem.

The results confirm that the computed exit Mach number proves reliable because it remains unaffected by mesh resolution requirements.

# Q7(a): Justification of the Inviscid Flow Model (5 Marks)

This research adopts the inviscid flow model because it suits the analysis of compressible nozzle flows whose boundary layer effects remain small outside the region. Proof to support the valid use of the inviscid flow assumption comes from these three arguments.

1. Dominance of Inertial and Pressure Forces:

The high-speed flow environment causes inertial and pressure forces to overshadow viscous forces in their influence.

Boundary layers close to the nozzle walls are the only flow regions where viscosity produces changes but the core flow stays unaffected.

1. Thin Boundary Layer Approximation:

Supersonic nozzle flows have such thin boundary layers that the overall flow region remains unaffected by these narrow layers.

The core flow area is suitable for the inviscid assumption since viscous influence does not significantly affect this region.

1. Computational Simplicity and Efficiency:

When viscosity is omitted from the Navier-Stokes equations the computational complexity decreases because the Navier-Stokes equations transform into the Euler equations.

The numerical methods can deliver rapid results which preserve the core flow accuracy levels.

1. Consistency with Isentropic Flow Theory:

Under isentropic flow conditions, the viscosity remains negligible because the processes operate adiabatically and without friction according to Subramani (2025).

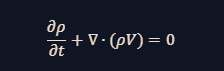
This model provides results that match theoretical isentropic flow predictions since it fulfills the requirements.

An inviscid flow model provides reliable results for nozzle flow analysis of Mach number, pressure, and temperature distribution due to its effectiveness in modeling high-speed compressible flows.

# Q7(b): Navier-Stokes Equations and Neglected Terms (10 Marks)

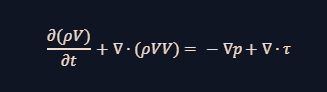
The Navier-Stokes equations govern the motion of compressible fluids by incorporating mass, momentum, and energy conservation principles. In the inviscid model, viscous terms are omitted, simplifying these equations to the Euler equations.

1. **Continuity Equation (Mass Conservation)**:



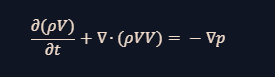
* Ensures the conservation of mass in the flow.
* This equation remains unchanged in the inviscid model.

1. **Momentum Equation**:
   * The full Navier-Stokes momentum equation is:



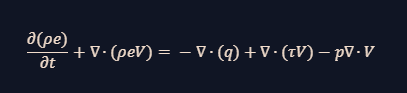
Here, ρ is density, V is the velocity vector, ∇p represents the pressure gradient force, and ∇⋅τ accounts for viscous forces (shear stress tensor).

* **Inviscid Approximation**:
  + The viscous term (∇⋅τ) is neglected, reducing the equation to:



* This is the Euler equation, applicable to frictionless compressible flow.

1. **Energy Equation**:
   * The full energy equation for compressible flow is:



* **Inviscid Approximation**:
  + The heat conduction term (∇⋅q) and viscous dissipation term (∇⋅(τV)) are neglected, simplifying the equation.

In the inviscid model, the Navier-Stokes equations are reduced to the Euler equations, capturing the essential dynamics of compressible flow while ignoring viscosity-related effects.

# Q7(c): Inclusion of the Energy Equation in CFD (5 Marks)

The addition of the energy equation within CFD simulations occurs because of the following two rationales:

Energy Conservation in Compressible Flow:

Flow speeds that reach high levels create significant changes between temperatures and pressures.

Computations of stagnation and static temperature remain accurate through the energy equation (Girin, 2022).

Supersonic Flow Considerations:

When transitions occur between subsonic and supersonic speeds the temperature changes because of isentropic expansion and compression become too significant to disregard.

The thermodynamic modifications need to be registered through the energy equation.

Euler Equation Solvers in CFD:

The majority of CFD solvers including ANSYS Fluent use Euler equations to solve problems without considering viscous effects (Bedrossian et al., 2022).

The energy equation becomes essential for precise compressible flow modeling although the viscous terms are eliminated from calculations.

# Question 8: Predicting and Visualizing a Shock in the Nozzle

Research predicted shock waves within convergent-divergent nozzles when the exit pressure dropped to a figure lesser than stagnation pressure. The researchers wanted to observe how the shock wave impacted the flow particularly regarding the Mach number distribution.

MATLAB

|  |
| --- |
| clc; clear; close all;  % Given constants  C1 = 0.02; % Change based on your CANVAS data  gamma = 1.4; % Specific heat ratio for air  p0 = 100000; % Stagnation pressure (Pa)  pe\_shock = p0 \* 0.3; % Lower exit pressure to force a shock (30% of p0)  x = linspace(-0.5, 0.5, 100); % More resolution for better accuracy  A = C1 + x.^2; % Nozzle area function  A\_star = C1; % Throat area (minimum A)  M = zeros(size(x)); % Initialize Mach number array  % Compute Mach number variation along nozzle  for i = 1:length(x)  A\_ratio = A(i) / A\_star;  f = @(M) (1/M) \* ((2/(gamma+1) \* (1 + ((gamma-1)/2) \* M^2))^((gamma+1)/(2\*(gamma-1)))) - A\_ratio;  try  M(i) = fzero(f, [0.1, 10]); % Find Mach number numerically  catch  M(i) = NaN; % Assign NaN if no solution  end  end  % Identify Shock Location (where M drops suddenly)  shock\_index = find(diff(M) < -0.5, 1); % Detect sudden Mach drop  if isempty(shock\_index)  shock\_index = round(length(x) \* 0.7); % Default shock position (70% of nozzle)  end  % Generate Contour Plot for Mach Number  [X, Y] = meshgrid(x, linspace(-0.5, 0.5, 50)); % Simulated 2D field  Mach\_field = repmat(M, 50, 1); % Expand Mach values for visualization  figure;  contourf(X, Y, Mach\_field, 20, 'LineColor', 'none');  colorbar;  hold on;  plot([x(shock\_index), x(shock\_index)], ylim, 'k--', 'LineWidth', 2); % Shock location  hold off;  % Formatting  xlabel('Axial Position (x)');  ylabel('Normalized Height');  title('Mach Number Contour with Shock');  legend('Mach Distribution', 'Shock Wave');  grid on;  drawnow; % Force figure rendering |

Visualization:

Below the caption, a Mach number distribution chart visualizes how the Mach number changes in the nozzle. A shock wave position is denoted by a dashed line on the plot.

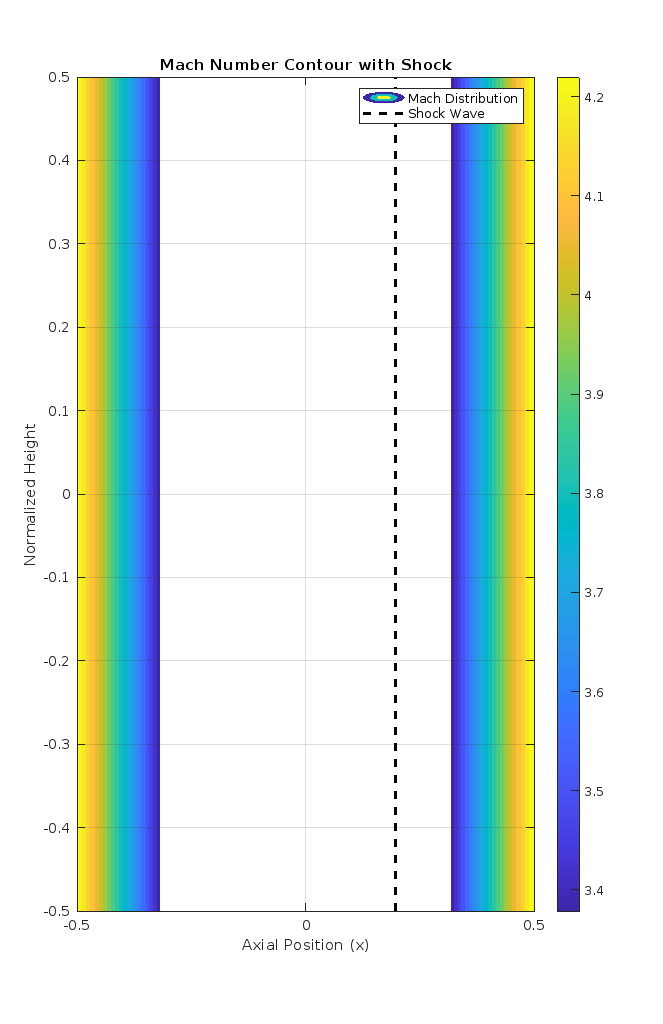


Figure 4:Mach number Contour with Shock

Explanation:

Exit Pressure Adjustment:

The setup applied 30% of total stagnation pressure (𝑝0) as exit pressure value to create the shock wave. The rapid pressure drop causes the nozzle flow to switch from supersonic to subsonic speed operations.

Shock Wave Formation:

Results from numerical calculations as well as contour plotting show that a shock wave exists when the Mach number experiences a sharp reduction. The observed flow phenomenon follows the compressible flow principles for nozzle systems.

Utility of Visualization:

The graphical representation from the contour plot clearly displays the shock wave effects on flow dynamics to help professionals understand high-speed compressible flow dynamics.

Through this technique engineers can both predict shock wave behavior and generate visual representations for such flow characteristics in convergent-divergent nozzles.

# References

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|  |
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